И. Хоменко, д-р филос. наук, проф.
Киевский национальный университет имени Тараса Шевченко, Киев, Украина

УРОВНИ И ИЗМЕРЕНИЯ АРГУМЕНТАЦИИ

Эта статья фокусируется на подходе, в котором различаются два уровня аргументации, связанные с объектными рассуждениями и мета-рассуждениями. Кроме этого, я различаю три измерения аргументации (логическое, диалектическое и риторическое). Уровни и измерения аргументации рассматриваются с точки зрения неформальной логики – дисциплины, находящейся на границе логического и эпистемологического знания. Я анализирую уровни и измерения аргументации с целью уточнения ключевых характеристик аргументативных рассуждений, которые являются предметом изучения неформальной логики.

Ключевые слова: неформальная логика, реальный аргумент, уровень аргументации, ряд аргументации, критический вопрос.

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Y. Shramko, Doctor of Science (Philosophy), Professor
Kryvyi Rih State Pedagogical University, Kryvyi Rih, Ukraine

SEMANTIC REPRESENTATION OF INCONSISTENT INTUITIONISTIC THEORIES

In this paper I propose a new method of semantic modeling for intuitionistic logic and provide an intuitive justification to this method. I put in the focus of consideration a concept of intuitionistic theory which is the basic concept of the whole analysis.

Keywords: Intuitionistic logic, state descriptions, constructive truth.

1. Intuitionistic theory

Originally intuitionism has been conceived and usually is treated as a special direction in the foundations of mathematics. Accordingly, one interprets intuitionistic logic as a logic of intuitionistic mathematics. Under this interpretation an intuitionistic theory can only be a mathematical theory, namely a mathematical theory constructed in accordance with the principles of intuitionism. However, one may try to extend the sphere of possible applications of these principles. Why not consider a possibility of a physical or chemical intuitionistic theory?[1] In what follows I understand under an intuitionistic theory *any* theory that fulfills some basic principles of intuitionism and is developed by means of intuitionistic logic. Among these principles are:

(1) interpreting truth as constructive provability (a sentence is intuitionistically true if and only if it is constructively proved);

(2) the principle of preservation for true propositions (a sentence once proved remains such in the future):

(3) rejection of the abstraction of actual infinity and acceptance of the abstraction of potential infinity.

One usually defines a theory as a set of sentences closed under the logical consequence. However, this definition is formulated within a paradigm of classical logic and does not correspond neither to intuitionistic concept of theoretic (scientific) activity nor to the above mentioned general principles of intuitionism. This definition presupposes evidently the abstraction of actual infinity and brings to naught the concept of truth as *constructive* provability.

Thus, it would be more suitable to define an intuitionistic theory as a set of sentences that *should* be closed under the logical consequence. That is, a sentence belongs to an intuitionistic theory (to some moment *a*) if and only if it is *actually* proved within this theory (to this moment). In this way we obtain a possibility to reflect the process of development of our knowledge and to distinguish between different stages of a theory.

2. The statements *of* a theory and the statements *about* a theory

Consider some intuitionistic theory. We should strongly distinguish between the statements of this theory itself and the statements by which we describe a state of the theory to some moment. This distinction corresponds to distinction between an object language and a metalanguage. Take some sentence formulated in the object language, say *A*. We have the following criterion – *A* belongs to our theory (to some moment) – *A* is true – if and only if *A* is proved within this theory (to this moment). Using the expressions of a metalanguage, we may describe the situation that takes place in the given theory. There are only two kinds of such expressions possible – either *positive* or *negative*. Namely, relative to any sentence *A* we may state either "*A* is proved in the given theory" or "*A* is not proved in the given theory".

Note that the negation in the later metadescription is not a negation of the object intuitionistic language. Moreover, this negation is essentially of classical character. The statements of the metalanguage do not obey generally the principle of truth-preservation, in particular the negative statements do not. In fact, a sentence can be not proved now, but the proof we need can be found later. Unlike this, the negative statements of intuitionistic theories should be of constructive type subject to the principles (1) – (3) above (and maybe some other principles).

Thus, we have *two* different kinds of negation – the object language negation which is applicable to the sentences of an intuitionistic theory and the metalanguage negation dealing with the statements by which we describe the theory. Consider our sentence *A* again. The object language (intuitionistic) negation of such a proposition has to be expressed in the form "*A* is refuted", or – as it is generally accepted in intuitionism – "assertion of *A* leads to a contradiction". A metalanguage negation of the proposition is, as against, simply "*A* is not proved".

3. The factual negation. Intuitionistic state-descriptions

The above distinction between two kinds of negation is a generalization of Heyting's distinction between "mathematical" and "factual" negations that can be found in [Heyting 1956]. Below is the full length corresponding passage from that work (italics are mine):

"Strictly speaking, we must well distinguish the use of 'not' in mathematics from that in explanations which are not mathematical, but are expressed in ordinary language. In mathematical assertions no ambiguity can arise: 'not' has always the strict meaning. 'The proposition *p* is not true', or 'the proposition *p* is false' means 'If we suppose the truth of *p*, we are led to a contradiction'. But if we say that the number-generator r which I defined a few moments ago is not rational, this is not meant as a mathematical assertion, but as *a statement about a matter of facts*; I mean by it that as yet no proof for the rationality of r has been given. As it

[1] Remember in this connection the "constructive theory of science" by P. Lorenzen and W. Kamlah ("Erlangener Schule").

is not always easy to see whether a sentence is meant as a mathematical assertion or as a *statement about the present state of our knowledge*, it is necessary to be careful about the formulation of such sentences. Where there is some danger of ambiguity, we express the mathematical negation by such expressions as 'it is impossible that', 'it is false that', 'it cannot be', etc., while *the factual negation* is expressed by 'we have no right to assert that', 'nobody knows that', etc.

There is a criterion by which we are able to recognize mathematical assertions as such. Every mathematical assertion can be expressed in the form: 'I have effected the construction *A* in my mind'. The mathematical negation of this assertion can be expressed as 'I have effected in my mind a construction *B*, which deduces a contradiction from the supposition that the construction *A* were brought to an end', which is again of the same form. On the contrary, *the factual negation* of the first assertion is: 'I have not effected the construction *A* in my mind'; this statement has not the form of a mathematical assertion." [Heyting 1956, 18-19].

To sum up: the *mathematical* negation by Heyting is the intuitionistic negation proper which can occur in intuitionistic theories. The *factual* negation is a metalanguage negation that belongs to a metalanguage used for describing intuitionistic theories.

Now I employ the principle of compositionality according to which any complex expression can be reduced to its constituents up to the very simple expressions. Taking as a philosophical postulate, this principle tells us that the world can be completely described on the level of atomic facts only. R. Carnap implemented this idea in semantic analysis by means of *state-descriptions* (see [Carnap 1988]). Applying this idea to the concept of intuitionistic theory, we may suppose that for a complete description of a state of some theory *a* to a certain moment, we can confine ourselves by listing all those atomic sentences that are proved in *a* to this moment as well as all those atomic sentences that are not proved in *a* to this moment.

In this way we arrive at the concept of intuitionistic state-description.

Let "**~**" be the negation of intuitionistic object language, and let "$\neg$" be factual negation used for describing the states of an intuitionistic theory.

Let **V** be the set of all atomic sentences of the language together with their factual negations: $\{p_1, \neg p_1, \ldots, p_n, \neg p_n, \ldots\}$.

Definition 3.1.
a is an *intuitionistic state-description* (i.s.d.) if and only if
(i) $a \subseteq \mathbf{V}$;
(ii) for any p_i: $p_i \in a$ or $\neg p_i \in a$.

If *a* is some i.s.d., then "$p_i \in a$" means "p_i is proved in the theory determined by *a*", and "$\neg p_i \in a$" means "p_i is not proved in the theory determined by *a*". Thus, any i.s.d. describes (on the level of atomic sentences) a state of some intuitionistic theory at some moment. Intuitionistic state-descriptions are "epistemic" *alter ego* of classical state-descriptions introduced by Carnap, with the difference that i.s.d. are descriptions of our knowledge rather than the "objective" world.

Generally the factual negation "$\neg$" as a component of i.s.d. is *not* equal to the negation of our metalanguage. Every occurrence of factual negation is of course an occurrence of metalanguage negation, but not *vice versa*! In general case metalanguage negation can be applied to any expression of the metalanguage, whereas the factual negation can be used only on the level of *facts* (i.e. can be applied only to atomic sentences). Thus, the factual negation "$\neg$" is a particular case of metalanguage negation. Taking into account the principle of compositionality, I suppose that such a factual negation is enough for complete description of any intuitionistic theory.

4. Two concepts of proof. Inconsistent state descriptions

Note, that condition (ii) of definition 3.1 demands that i.s.d. must be *complete* with respect to factual negation. Now the problem of inconsistency arises: it is *not* generally required that i.s.d. have to be *consistent* with respect to factual negation, that is, I *do not* take the condition

(iii) $p_i \notin a$ or $\neg p_i \notin a$.

This may seem to be very strange: taking into account the underlying intuitive interpretation, this means that a situation can appear when some sentence is and is not proved simultaneously. How can it be? Some primary intuitive ideas – first of all the law of contradiction – seems to be afforded. I believe however that this situation can be explained in an intuitively satisfactory way, and this can be done just in accordance with the Heyting's understanding of mathematical (object language) and factual (metalanguage) negations, and my interpretation proposed above.

For the sake of simplicity I confine myself with a consideration of axiomatic theories. Let us first spell out the meaning of the expression "$p_i \in a$". It means that a proof of p_i (in theory *a*) is given, that is – according to the tradition – there is a sequence of sentences such that any sentence from the sequence is either axiom of *a*, or is obtained by inference rules, and the last sentence of the sequence is p_i.

Now, if we wish that i.s.d. describe real intuitionistic theories, we have to take into consideration the fact of existence of *inconsistent theories*. This fact simply takes place, our theories – regardless whether we wish this or not – can be and often really are inconsistent. In this light the following question arises:

What is the proper semantic representation of the situation, when a sentence is proved within an inconsistent theory?

Let us take the following definition: an intuitionistic theory is inconsistent if and only if there is a sentence *A*, such that *A* is proved in it, and *~A* is proved in it. Consider now a theory which is inconsistent with respect to p_i. That is, the proofs of both p_i and $\sim p_i$ in the theory are given: there is a sequence of sentences such that any sentence from the sequence is either axiom of a theory, or is obtained by an inference rule, and the last sentence of the sequence is p_i, and there is a sequence of sentences such that any sentence from the sequence is either axiom of a theory, or is obtained by an inference rule, and the last sentence of the sequence is $\sim p_i$. But the last observation means that *in fact* p_i is not proved, i.e. that the above mentioned "proof" (sequence of sentences) for p_i *proves nothing*! However, this "proof" *is still present* in our theory (as long as our theory is contradictory). Thus, we have an interesting metatheoretical situation – *formally* we have a proof of p_i, but this proof does not prove p_i, so, *actually*, we do not have a proof of p_i.

This argument can be easily reconstructed so that it does not contain any explicit reference to the object-language negation. Let us take (as Heyting does) the notion of contradiction as a primitive notion, and let us define a contradictory theory as a theory that includes some self-contradictory sentence (e.g., 1 = 2) as a theorem. Again, if a sentence p_i is proved in such a theory, then, of course, we do have a formal proof of p_i, but nevertheless, we cannot seriously state that p_i is really proved (because the theory, where the proof of p_i is given, is contradictory and as such incredible).

We should clearly distinguish between two different meanings of the expression "p_i is proved" – the *merely*

formal one and the *real* one. From a formal point of view to say "p_i is proved" means to say "there is a sequence of sentences such that ... etc." (as above) and nothing more than that. But if an intuitionist says "p_i is proved" (having in mind a real meaning of the term) she/he means that p_i is *intuitionistically true* (and this of course cannot be the case, when p_i is self-contradictory).

This distinction can also be explicated as a distinction between a *weak* notion of proof (a formal proof in the theory is given), and a *strong* notion of proof (a formal proof in the theory is given, *and* the theory is consistent). Correspondingly, the expression "p_i is not proved" can be used in two senses: either (1) there is no formal proof of p_i in the given theory, or (2) there is no formal proof of p_i in the given theory, or the theory is inconsistent. The second meaning reflects an understanding that *any formal proof of p_i in a contradictory theory cannot be considered a real proof of p_i.*

Using the terminology of i.s.d., I interpret (1) as $p_i \notin a$, and (2) as $\neg p_i \in a$. It is clear that (1) $\Rightarrow$ (2), but not *visa versa*! This is why I take the condition (ii) but do not take (iii). I also interpret the expression $p_i \in a$ as "there is a formal proof of p_i in theory a".

Taking into account either absence or presence of a formal proof of p_i in the theory a, and either consistency or inconsistency of a itself, the following four situations are possible:

I. (a) there is no formal proof of p_i in theory a, and a is inconsistent;

(b) there is no formal proof of p_i in theory a, and a is consistent.

II. (a) there is a formal proof of p_i in theory a, and a is inconsistent;

(b) there is a formal proof of p_i in theory a, and a is consistent.

As noted above, if we have no formal proof of a sentence, then the real proof of the sentence is absent anyway, regardless whether the theory is consistent or not. That is, I(a) and I(b) can be interpreted as single case. However, if we have a formal proof of a sentence, we have to take into account the factor of consistency of the theory to get to know whether the sentence is really proved or not. It means that the cases II(a) and II(b) have to be interpreted separately. All these cases can be adequately described by means of combinations of expressions $p_i \in a$, $\neg p_i \in a$, $p_i \notin a$ and $\neg p_i \notin a$ alone. Here are these descriptions:

I(a) – (b): If there is no formal proof of p_i in theory a, so it means that there is no proof of p_i in a at all, hence, $p_i \notin a$ and $\neg p_i \in a$.

II(a): p_i is formally proved in a, but not really. Thus, there is a formal proof of p_i in a, however, there is no real proof of p_i in a: $p_i \in a$ and $\neg p_i \in a$. Then we have: there is a formal proof of p_i in a, and (there is no formal proof of p_i in a, or a is inconsistent). As a result, using the rule of *disjunctive syllogism*, we get – "the theory a is inconsistent"! So, contradictory i.s.d. $\{p_i, \neg p_i\}$ serves as the adequate semantic representation of a contradictory intuitionistic theory.

II(b): p_i is really proved in the theory a: $p_i \in a$ and $\neg p_i \notin a$.

I summarize these descriptions in the following table:

p_i is not formally proved in a	there is no formal proof of p_i in theory a	$p_i \notin a$ and $\neg p_i \in a$; $a = \{\neg p_i\}$
p_i is formally proved in a, but not really	there is a formal proof of p_i in theory a, and a is inconsistent	$p_i \in a$ and $\neg p_i \in a$; $a = \{p_i, \neg p_i\}$
p_i is really proved in the theory a	there is a formal proof of p_i in theory a, and a is consistent	$p_i \in a$ and $\neg p_i \notin a$; $a = \{p_i\}$

One may notice that *in fact* contradictory i.s.d. – under such an interpretation – are not contradictory at all. Yes, they are not. I would like to stress that "contradictory" i.s.d. are not contradictory itself, they only *represent* the contradictory theories. An i.s.d. a would have been *really* contradictory, if we would have $p_i \in a$ and $p_i \notin a$. But this is impossible, because the whole semantic construction would have turned then into nonsense. Introduction of factual negation helps to solve a sophisticated technical problem – to represent inconsistent theories in a non-contradictory way. In other words, the factual negation proves to be a very suitable technical tool for representing inconsistent theories on the semantic level. Contradictory i.s.d. provide a construction where the strong and the weak concepts of proof can be combined. If we have $\{p_i, \neg p_i\}$, it simply means: "Although we have a formal "proof" of p_i, nevertheless p_i is not true (because the theory, where the "proof" was given, is contradictory)".

Notice, that the interpretation of factual negation given above perfectly corresponds to Heyting's understanding of the factual negation. Heyting writes that factual negation can be expressed as "we have no right to assert that". But this is exactly the case (2) described above – we have no right to assert p_i if and only if either no formal proof of p_i is given or such a proof is given in a contradictory theory.

5. A general model for intuitionistic theories

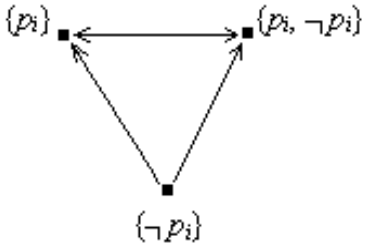
The definition of intuitionistic theory given in the section 1 implies that we should be able to observe a development of our theoretical knowledge in the course of time. The apparatus of i.s.d. gives an excellent opportunity for reflecting this idea. Namely, we may introduce a binary relation R between intuitionistic states-descriptions as follows:

Definition 5.1.

R$ab \Leftrightarrow a^+ \subseteq b^+$ [a^+ (b^+) is the "positive" part of a (b), i.e. a^+ (b^+) is that and only that part of a (b) which consists of the variables without metanegations].

Informally relation R can be interpreted as a possible time-relation between different states of some theory, i.e., Rab means that theoretical construction b is a result of possible development of theoretical construction a. It is easy to see that R is reflexive and transitive.

With respect to every atomic sentence p_i only the following three i.s.d. are possible: $\{\neg p_i\}$, $\{p_i\}$, $\{p_i, \neg p_i\}$. These i.s.d. are ordered by the relation R as follows:



Now I introduce a *general model for intuitionistic logic* (G-model), on the base of intuitionistic state-descriptions as a triple < W, R, ||– >, where W is a non-empty set of i.s.d.,

R is a binary relation on W defined as above, and ||– is a *forcing relation* between i.s.d. and formulae of the language. The expression "*a* ||– *A*" means "i.s.d. *a forces us* to accept the sentence *A*", or according to the informal explanations given above "the sentence *A* is proved within a theory determined by *a*". The following definition introduces the forcing relation for atomic and positive complex sentences:

Definition 5.2.
$a \Vdash p_i \Leftrightarrow p_i \in a$;
$a \Vdash A \,\&\, B \Leftrightarrow a \Vdash A$ and $a \Vdash B$;
$a \Vdash A \vee B \Leftrightarrow a \Vdash A$ or $a \Vdash B$;
$a \Vdash A \supset B \Leftrightarrow \forall b\,(Rab \Rightarrow (b \Vdash A \Rightarrow b \Vdash B))$.

One can easily show that the principle of truth-preservation holds both for atomic and for complex sentences.

Finally, I would like to point out that inconsistent intuitionistic state-descriptions are not only of pure theoretic interest, but can be effectively employed in some key semantic definitions. A remarkable feature of the semantic model proposed above, is that it allows to define intuitionistic negation in a very natural way. As it was already mentioned at the beginning of the paper, the traditional approach to informal understanding of the negation operation in intuitionism is that any sentence *~A* can be considered true if and only if an assumption that *A* is true leads us to contradiction. Exactly such an understanding is presented in the citation from [Heyting 1956] above. Thus, the operation of negation is reduced to the notion of contradiction. Heyting wrote that "contradiction must be taken as a primitive notion. It seems very difficult to reduce it to simpler notions..." [Heyting 1956, 98]. However, it appears that constructing semantics in terms of intuitionistic state-descriptions, allows to introduce the notion of contradiction *by definition*. Let "con(*a*)" means "intuitionistic state description *a* is contradictory". Then we may consider the following definition:

Definition 5.3.
$\mathrm{con}(a) \Leftrightarrow \exists p_i\,(p_i \in a$ and $\Box p_i \in a)$.

Now the forcing relation for negation can be defined as follows:

Definition 5.4.
$a \Vdash {\sim}A \Leftrightarrow \forall b\,(Rab \Rightarrow (b \Vdash A \Rightarrow \mathrm{con}(b)))$.

This definition *literally reproduces* the informal understanding of negation operator described above. It is also interesting that by means of this definition the *minimal negation* of [Johansson 1936] is adequately defined. To obtain the negation of Heyting's intuitionistic logic, we have to introduce the notion of absolute contradictory i.s.d. *a* (abcon(*a*)):

Definition 5.5.
$\mathrm{abcon}(a) \Leftrightarrow \forall p_i\,(p_i \in a$ and $\neg p_i \in a)$,

and to accept the following condition:

Condition 5.6.
$\mathrm{con}(a) \Rightarrow \mathrm{abcon}\,(a)$

This condition validates the characteristic axiom of intuitionistic logic:

EFQ. ~A ⊃ (A ⊃ B).

Taking further conditions we may get the definitions of other negations of intuitionistic type, cf. [Shramko 1997a], [Shramko 1997b].

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Я. Шрамко, д-р філос. наук, проф.
Криворізький державний педагогічний університет, Кривий Ріг, Україна

СЕМАНТИЧНА РЕПРЕЗЕНТАЦІЯ НЕСУПЕРЕЧЛИВИХ ІНТУЇЦІОНІСТСЬКИХ ТЕОРІЙ

У цій статті я пропоную новий метод семантичного моделювання для интуиционистской логіки і забезпечую інтуїтивне обгрунтування цього методу. Я поставив в центр уваги концепцію интуиционистской теорії, яка є базовою концепцією всього аналізу..

Ключові слова: Інтуїціоністська логіка, опис стану, конструктивна істина.

Я. Шрамко, д-р филос. наук, проф.
Криворожский государственный педагогический университет, Кривой Рог, Украина

СЕМАНТИЧЕСКОЕ ПРЕДСТАВЛЕНИЕ ПРОТИВОРЕЧИВЫХ ИНТУИЦИОНИСТСКИХ ТЕОРИЙ

В этой статье я предлагаю новый метод семантического моделирования для интуиционистской логики и обеспечиваю интуитивное обоснование этому методу. Я поставил в центр внимания концепцию интуиционистской теории, которая является базовой концепцией всего анализа.

Ключевые слова: Интуиционистская логика, описание состояния, конструктивная истина.